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THE PHENOMENON OF VIBRATION TRANSFER IN TWO-COMPONENT OSCILLATING INTERACTING SYSTEMS

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We consider the phenomenon of vibration transfer of a physical quantity described by a scalar parameter and obeying the law of conservation in a two-component interacting medium. Expressions for the vibration-transfer coefficient are obtained for various cases of practical interest. A phantom model of vibration transfer is suggested. The results of a numerical experiment are discussed.

It is well known that vibrational motion of the components in multicomponent interacting systems gives rise to new phenomena of transfer [1]. However, the basic laws governing these phenomena have been studied insufficiently well. This hinders the practical application of the phenomenon of vibration transfer in engineering devices, for example, in thermal elements with controlled thermal conductivity. Moreover, the development of the theory of this process is of general scientific importance for deriving turbulence models [2].

In the present paper we develop a model for vibration transfer of scalar physical characteristics for which conservation laws are satisfied. In this case, diffusion-type processes are ignored and only the convective component of transfer is taken into consideration.

The investigations are based on the conservation laws that in the form of a continuity equation are used for describing interacting physical systems. Generally, the interaction of systems is expressed by nonlinear relations. In the particular case when there are small deviations of local parameters from equilibrium values, Newton's linear law is used. For simplicity we assume the presence of two interacting components moving according to prescribed laws that correspond to the autonomy conditions. In addition, we consider the evolution of the scalar local s and integral S parameters (mass, charge, etc.).

The evolution of the local parameter s in a two-component medium is described by the equations:

$$\frac{\partial s_1}{\partial t} + U_1(t)\frac{\partial s_1}{\partial x} + \alpha_1 (s_1 - s_2) = 0\,, \tag{1}$$

$$\frac{\partial s_2}{\partial t} + U_2(t)\frac{\partial s_2}{\partial x} + \alpha_2 (s_2 - s_1) = 0\,, \tag{2}$$

where $U_1(t)$ is the velocity of the i-th component ($i = 1, 2$), $\alpha_i (> 0)$ is the coefficient of the interaction of systems.

We note that system (1)-(2) admits an exact solution in the case of constant gradients

$$\frac{\partial s_1}{\partial x} = \frac{\partial s_2}{\partial x} = -\Gamma\,. \tag{3}$$

Under the conditions of the equality of s_1 and s_2 at the initial time instant the solution takes on the form:

$$s_1 = -\Gamma x + \Gamma \int_0^t U_1(\tau)\,d\tau + \alpha_1 \Gamma \int_0^t d\tau \int_0^\tau U(t')\exp(\alpha(t' - \tau))\,dt'\,, \tag{4}$$

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$$s_2 = -\Gamma x + \Gamma \int_0^t U_2(\tau)\, d\tau - \alpha_2 \Gamma \int_0^t d\tau \int_0^\tau U(t') \exp(\alpha(t' - \tau))\, dt' , \tag{5}$$

where $U(t) = U_2(t) - U_1(t)$; $\alpha = \alpha_1 + \alpha_2$.

In the case of harmonic oscillations $U_1 = A_1\omega_1 \cos \omega_1 t$ and $U_2 = A_2\omega_2 \cos(\omega_2 t + \varphi)$ the expression for the local parameters is as follows:

$$\begin{gathered} s_1 = -\Gamma x + \Gamma A_1 \sin \omega_1 t + \alpha_1 \Gamma I ; \\ s_2 = -\Gamma x + \Gamma A_2 \left[\sin(\omega_2 t + \varphi) - \sin \varphi\right] - \alpha_2 \Gamma I ; \end{gathered} \tag{6}$$

$$I = \frac{A_2\omega_2\alpha}{\alpha^2 + \omega_2^2}\left[\frac{\sin(\omega_2 t + \varphi)}{\omega_2} - \frac{\sin\varphi}{\omega_2} - \frac{\cos(\omega_2 t + \varphi)}{\alpha} + \frac{\exp(-\alpha t)\cos\varphi}{\alpha} - \right.$$

$$\left. - \frac{\omega_2(1 - \exp(-\alpha t))\sin\varphi}{\alpha^2}\right] - \frac{A_1\omega_1\alpha}{\alpha^2 + \omega_1^2}\left[\frac{\sin\omega_1 t}{\omega_1} - \frac{\cos\omega_1 t - 1}{\alpha} - \frac{1 - \exp(-\alpha t)}{\alpha}\right].$$

When the gradients of the local parameters differ from constant ones, difficulties arise in obtaining analytical solutions.

The transfer of the integral parameter S, corresponding to the local parameter s, occurs along the x axis. This transfer is carried out due to a convective process. The magnitude of the flux of the parameter s

$$j = \delta s U , \tag{7}$$

where δ is a constant coefficient, generally contains oscillating components. The mean value of the convective flow, without allowance for the interaction $(\alpha_1 = 0)$, for the period of vibrations

$$\langle j \rangle = \delta \langle sU \rangle \tag{8}$$

is equal to zero in the case of harmonic oscillations. However, when $\alpha_1 \neq 0$, an additional transfer of the local parameter s appears, which is called vibration transfer.

Now, we shall calculate the effective coefficient of vibration transfer. For this purpose, we find the value of Θ:

$$\Theta = \frac{k_1}{l} \int_0^l (s_1 - s_2)\, dx = \frac{k_1}{l} \int_0^t \exp(\alpha(\tau - t)) \int_0^l \frac{\partial}{\partial x}(U_2 s_2 - U_1 s_1)\, dx d\tau , \tag{9}$$

where $k_1 = k\alpha$. In the case of constant gradients

$$\Theta = -k_1 \Gamma \int_0^t U(\tau) \exp(\alpha(\tau - t))\, d\tau . \tag{10}$$

The magnitude of the flux of the parameter s is equal to

$$j = -\langle \Theta x \rangle = k_1 \Gamma \left\langle \left(\int_0^t \exp(\alpha(\tau - t))\, U(\tau)\, d\tau \right) \left(\int_0^t U(\tau)\, d\tau \right) \right\rangle .$$

The effective vibration-transfer coefficient is determined by averaging the flux [3]

$$\kappa = -k_1 \left\langle \left(\int_0^t \exp(\alpha(\tau - t))\, U(\tau)\, d\tau \right) \left(\int_0^t U(\tau)\, d\tau \right) \right\rangle . \tag{11}$$

The calculation of the integrals entering into Eq. (11) does not involve particular complexities in spite of considerable numerical effort. In the case of harmonic oscillations the quantity Θ is represented as (for large t)

$$\Theta = k_1\Gamma \left[\frac{A_1}{\sqrt{1+\Phi_1^2}} \sin(\omega_1 t + \varphi_1) - \frac{A_2}{\sqrt{1+\Phi_2^2}} \sin(\omega_2 t + \varphi + \varphi_2)\right], \tag{12}$$

where $\Phi_1 = \alpha/\omega_1$; $\Phi_2 = \alpha/\omega_2$; $\tan\varphi_1 = \alpha/\omega_1$; $\tan\varphi_2 = \alpha/\omega_2$. The expression to be averaged seems to be very unwieldy. The result of averaging depends on the relationship between ω_1 and ω_2.

At large differences between the frequencies $(\omega_2 - \omega_1)/\omega_1 >> 1$ or $(\omega_1 - \omega_2)/\omega_2 >> 1$, we obtain

$$\kappa = \frac{k_1 A_1^2}{2} \frac{1}{1+\Phi_1^2} + \frac{k_1 A_2^2}{2} \frac{1}{1+\Phi_2^2}. \tag{13}$$

When the frequencies $\omega_1 = \omega_2 = \omega$ coincide, the result of averaging depends on the phase difference φ:

$$\kappa = \frac{k_1 (A_1^2 + A_2^2)}{2(1+\Phi^2)} - \frac{k_1 A_1 A_2}{1+\Phi^2} \cos\varphi. \tag{14}$$

When $\varphi = 0$, Eq. (14) yields

$$\kappa = \frac{k_1 (A_1 - A_2)^2}{2(1+\Phi^2)}. \tag{15}$$

At equal amplitudes $A_1 = A_2 = A$

$$\kappa = \frac{k_1 A^2}{1+\Phi^2} (1 - \cos\varphi). \tag{16}$$

Thus, the vibration-transfer coefficient κ differs from zero, whereas regular convective heat transfer along the x axis is absent. If the components oscillate in opposite phase $\varphi = \pi$, then the vibration-transfer coefficient is $\kappa = 2k_1A^2/(1 + \Phi^2)$, and according to Eq. (16) in the case of identical oscillation phases $\varphi = 0$ vibration transfer is absent. Expressions (13)-(16) admit a simple physical interpretation. We note that the value of the vibration-transfer coefficient depends on the square of the amplitude of oscillations. At small frequencies $\omega_1 \to 0$, $\omega_2 \to 0$ the vibration-transfer coefficient is represented by a linear combination of the squared frequencies:

$$\kappa = \frac{k_1 A_1^2 \omega_1^2}{2\alpha^2} + \frac{k_1 A_2^2 \omega_2^2}{2\alpha^2}. \tag{17}$$

At large values of frequencies, the vibration-transfer coefficient can reach a limiting value:

$$\kappa = \frac{k_1 (A_1^2 + A_2^2)}{2}. \tag{18}$$

The phenomenon of vibration transfer can form a basis for creating new engineering devices (for example, thermal elements with controllable thermal conductivity) and explain previously discovered physical effects (for example, the increase in transfer coefficients in acoustic fields). We discussed the idea of using vibration transfer for supplying heat from the active zone of nuclear reactors without removal of a radioactive heat-transfer agent.

The investigation of system (1)-(2) is also of interest from another viewpoint. All the physical processes described by different terms in Eqs. (1) and (2) are reversible. At the same time, the process of vibration transfer considered above does not change sign on replacement of U_1 and U_2 by $-U_1$ and $-U_2$. This means that the "irreversibility" degree of vibration transfer increases as compared to initial physical effects. It is extremely important to understand the reason for this phenomenon, since this will make it possible to suggest new approaches in the theory of a random behavior of the system.

From Eqs. (1) and (2) we obtain two independent equations for defining s_1 and s_2:

$$\frac{\partial^2 s_1}{\partial t^2} + (U_1 + U_2)\frac{\partial^2 s_1}{\partial x \partial t} + U_1 U_2 \frac{\partial^2 s_1}{\partial x^2} + (\alpha_1 + \alpha_2)\frac{\partial s_1}{\partial t} + (\alpha_1 U_2 + \alpha_2 U_1)\frac{\partial s_1}{\partial x} = 0\,, \tag{19}$$

$$\frac{\partial^2 s_2}{\partial t^2} + (U_1 + U_2)\frac{\partial^2 s_2}{\partial x \partial t} + U_1 U_2 \frac{\partial^2 s_2}{\partial x^2} + (\alpha_2 + \alpha_1)\frac{\partial s_2}{\partial t} + (\alpha_2 U_1 + \alpha_1 U_2)\frac{\partial s_2}{\partial x} = 0\,. \tag{20}$$

The type of the equations is determined by the sign of the discriminant

$$D = (U_1 + U_2)^2 - 4U_1 U_2 = (U_1 - U_2)^2\,. \tag{21}$$

For harmonic oscillations $U_1 = A_1\omega_1 \cos(\omega_1 t + \varphi_1)$ and $U_2 = A_2\omega_2 \cos(\omega_2 t + \varphi_2)$ the discriminant is positive but at individual points it takes the value 0, so that system (1)-(2) is of the hyperbolic type with parabolic degeneration. The equation for characteristics

$$\left(\frac{dx}{dt}\right)^2 + (U_1 + U_2)\frac{dx}{dt} + U_1 U_2 = 0 \tag{22}$$

can be solved by either of two techniques: representation in the form of cofactors yields

$$\frac{dx}{dt} = -\,U_1(\tau)\,,\ \frac{dx}{dt} = -\,U_2(\tau)\,, \tag{23}$$

and by completing the square, we obtain

$$\frac{dx}{dt} = \frac{U_1 + U_2}{2} \pm \frac{|U_1 - U_2|}{2}\,. \tag{24}$$

We note that Eqs. (23) and (24) coincide when the difference $U_1 - U_2$ does not change sign. Otherwise, their integration leads to different results. Specifically, in the case of periodic functions, the solutions of Eqs. (23) are periodic, whereas the solutions of Eq. (24) are increasing functions $x(t)$.

Figure 1 gives the dependences of the characteristic velocities on time plotted according to Eq. (24) for various cases of harmonic oscillations.

In case *a*, when one of the components is motionless ($\omega_2 = 0$), the "exchange of characteristics" occurs at the points of the equality of characteristic velocities (23). The phenomenon of the "exchange of characteristics" leads to multiplication of the ways for evolution of the system. Although the number of families of characteristics in the system is equal to two, the number of the ways for the evolution of the system depends on the time $2^N = 2^{[2t/T]}$, where N is the number of the points at which characteristic velocities intersect. The "multiplication" of the ways of evolution leads to a situation in which the description of the system becomes less definite with time. This corresponds to the "onset of chaos" in the system, so that the arrangements described by Eqs. (1) and (2) can be called in this sense linear "generators of chaos." The presence of an infinite denumerable set of the points of intersection of characteristic velocities is a condition for the onset of chaos.

In other cases of the motion of components the "exchange of characteristics," i.e., the multiplication of the ways of evolution, seems to be more complex (see Fig. 1, cases b, c, d).

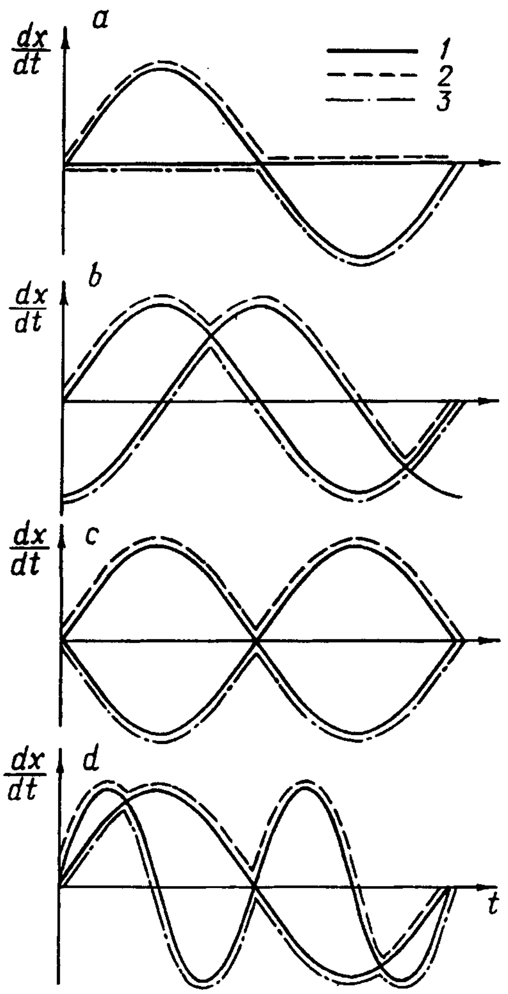

Fig. 1. The dependence of the characteristic velocity on time: a) one of the components is at rest, $\omega_1 = 0$; b) coincidence of frequencies, $\varphi = \pi/2$; c) the same, $\varphi = \pi$; d) multiplicity of the frequencies is equal to two, $\omega_1 = 2\omega_2$, $\varphi = 0$; 1) basic families of the characteristics; 2) upper "exchange" characteristic; 3) lower "exchange" characteristic.

We note that the characteristics obtained from Eq. (24) correspond to the motion of particles in a system of phantoms. The mechanism of transfer of the integral parameter S can be described by means of the motion of phantoms.

Let us calculate the "energy" of a phantom. For this purpose we assume that $\omega_2 = 0$ and $\omega_1 = \omega$, then from Eq. (12) we obtain

$$\Theta = k_1 \Gamma \frac{A}{\sqrt{1 + \Phi^2}} \sin(\omega t + \varphi_1) . \tag{25}$$

The magnitude of the phantom "energy" is the amount of the integral parameter S, flowing for the time interval, equal to a half period, along the x-axis length, equal to the double amplitude:

$$E_s = 2A \left| \int_{\varphi_1}^{\varphi_1 + T/2} \Theta(\tau)\, d\tau \right| = \frac{4A^2 k_1 \Gamma}{\sqrt{1 + \Phi^2}} = \frac{4A^2 k_1 \Gamma}{\sqrt{\alpha^2 + \omega^2}} \omega . \tag{26}$$

In systems with rigid interaction $\alpha \gg \omega$ the magnitude of the phantom "energy" is proportional to the frequency:

$$E_s = \frac{4A^2 k_1 \Gamma}{\alpha}\omega = h_s \omega\,, \quad h_s = \frac{4A^2 k_1 \Gamma}{\alpha}\,. \tag{27}$$

It is clear that the concept of phantoms is meaningful if the time intervals considered are larger than a half period $\Delta t > T/2$. Therefore, the condition for phantom observation will be written down in the form

$$E_s \Delta t > h_s \omega \frac{T}{2} = h_s \pi = \frac{4\pi k_1 \Gamma A^2}{\alpha}\,. \tag{28}$$

Relations (27) and (28) are analogs of the classical relations for photon energy and of the relation for the Heisenberg uncertainties.

If the gradients of the local parameters differ from constant ones, then it is difficult to obtain analytical parameters. This leads to the problem of numerical simulation of a multicomponent system. This case, however, presents basic difficulties. The computational process is determined and unambiguous, and the behavior of the system, even in a two-component case, is multivariant. Branching of the process at a macrolevel occurs (i.e., branching of the values of the integral parameter in time). In such a case a numerical model gives only one of the variants of the system behavior, and it remains only to hope that all the cases of the behavior at different values of the parameters are invariant, i.e., they are similar in a sense and reflect the general tendencies of system behavior.

As a basis for numerical simulation, we take the relationships in a two-component case:

$$s_1(x, (k+1)\Delta t) \approx s_1(x - \Delta t U_1(k\Delta t)\,, k\Delta t) + \Delta t \alpha_1 (s_2(x, k\Delta t) - s_1(x, (k\Delta t)))\,,$$

$$s_2(x, (k+1)\Delta t) \approx s_2(x - \Delta t U_2(k\Delta t)\,, k\Delta t) + \Delta t \alpha_2 (s_1(x, k\Delta t) - s_2(x, (k\Delta t)))\,,$$

using which from the values of s_1 and s_2 at the instant $k\Delta t$ we find the values of s_1 and s_2 at the time instant $(k+1)\Delta t$. The relationship is easily generalized for the case of three, four, or more components. In the two-component case the correctness of the approximate equalities follows from the fact that if the functions s_1 and s_2 satisfy the system

$$s_1(x, t + \varepsilon_1) = s_1(x - U_1(t)\,\varepsilon_1, t) + \varepsilon_1 \alpha_1 (s_2 - s_1) + o_1(\varepsilon_1)\,,$$

$$s_2(x, t + \varepsilon_2) = s_2(x - U_2(t)\,\varepsilon_2, t) + \varepsilon_2 \alpha_2 (s_1 - s_2) + o_2(\varepsilon_2)\,,$$

then they also satisfy system (1)-(2). Actually

$$\frac{\partial s_1}{\partial t} = \lim_{\varepsilon \to 0} \frac{s_1(x, t+\varepsilon) - s_1(x, t)}{\varepsilon} =$$

$$= \lim_{\varepsilon \to 0} \frac{s_1(x - \varepsilon U_1(t), t) + \varepsilon \alpha_1 (s_2 - s_1) - S_1(x, t) + o_1(\varepsilon)}{\varepsilon} =$$

$$= \lim_{\varepsilon \to 0} \frac{s_1(x - \varepsilon U_1(t), t) - s_1(x, t)}{\varepsilon} + \alpha_1 (s_2 - s_1) =$$

$$= \lim_{\varepsilon \to 0} \frac{s_1(x - \varepsilon U_1(t), t) - s_1(x, t)}{\varepsilon(-U_1(t))}(-U_1(t)) + \alpha_1 (s_2 - s_1) =$$

$$= \lim_{\varepsilon \to 0} \frac{s_1(x + \Delta x, t) - s_1(x, t)}{\Delta x}(-U_1(t)) + \alpha_1 (s_2 - s_1) =$$

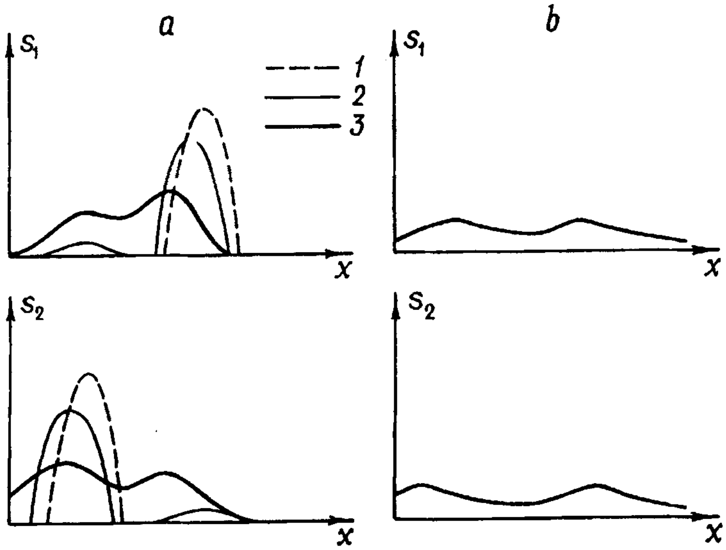

Fig. 2. Space-time distribution of local parameters s_1 and s_2 according to the results of computational experiment: a) for time $t = 0, 3, 7.5$ (curves 1, 2, 3); b) the same for large times ($t = 23$).

$$= -\frac{\partial s_1}{\partial x} U_1(t) + \alpha_1 (s_2 - s_1) .$$

The substitution into the initial system yields an identity.

Thus, we determine the values of s_1 and s_2 at the time instants 0, Δt, $2\Delta t$, ... , $k\Delta t$, $(k + 1)\Delta t$.

The choice of the branching variant depends on whether the value of $k\Delta t$ occurred at a branching point or not, as well as on the value of the parameter s at this point.

Figure 2 presents one of the variants of system evolution obtained from the results of calculations on an IBM computer.

Thus, the evolution of two-component interacting systems under the conditions of vibrational motion of components leads inevitably to phenomena of vibration transfer of a scalar parameter. The front of perturbations moves with a finite velocity with "multiplication" of the ways of evolution, and this increases the uncertainty in describing system. One of the methods for describing the system evolution is the phantom one. Here the energy of a phantom in systems with rigid interaction is proportional to frequency. Many of the conclusions of the present work can be easily extended to the evolution of systems with a large number of components and with transfer of vector integral parameters.

NOTATION

S_i, s_i, integral and local scalar parameters; t, time; x, spatial coordinate; U_i, velocity of i-th component; Γ, spatial gradient of scalar parameters; α_i, interaction coefficient; A_i, ω_i, φ_i, amplitude, frequency, and initial phase of oscillations; j, flux of the scalar parameter; κ, coefficient of vibration transfer; E_s, energy of phantom; ε_i, time step; $o_i(\varepsilon)$, infinitesimal; h_s, constant.

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